AP Precalculus Practice Free-Response 1

x	1	2	4	8	16
$f(x)$	0	-7	-14	-21	-28

1. (Calculator) Selected values of a function f are given in the table above. It is known that f is a strictly decreasing function defined for $x > 0$. Let the function g be given by

$$g(x) = \frac{3x^2 - 14x - 5}{x - 6}$$

(a) i. Let the function h be defined by $h(x) = (g \circ f)(x)$. Find the value of $h(1)$, or indicate that it is not defined.

ii. Find the value of $f^{-1}(-7)$ or indicate that it is not defined.

(b) i. Find all values of x such that $g(x) = 15$, or indicate that there are no such values.

ii. Find all vertical asymptotes, horizontal asymptotes, and slant asymptotes of g, if they exist.

(c) i. Use the table of values for f to determine if f is best modeled by a linear, quadratic, cubic, exponential, or logarithmic function.

ii. Justify your answer to the previous question based on the relationship betwen the change in the output values of f and the change in the input values of f.

2. (Calculator) The water level in a city's reservoir is modeled by the function W, given by

$$W(t) = b + 32a^t$$

where $W(t)$ is measured in meters, and t is measured in hours elapsed since midnight ($t = 0$). At midnight, the water level of the reservoir is 70 meters, and at 8 a.m., the water level has dropped to 45 meters.

(a) Using the given initial data, find the values of constants a and b in the expression for $W(t)$.

(b) i. Find the average rate of change of the water level, in meters per hour, from $t = 0$ to $t = 8$. Show the computations that lead to your answer.

ii. Use your answer in part i. to estimate the water level in the reservoir at 4 a.m. Show the work that leads to your answer.

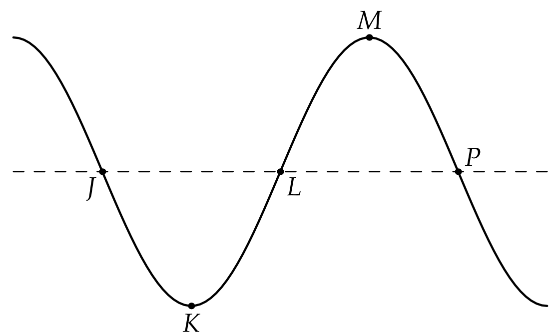
iii. Is every approximation of the water level from $t = 0$ to $t = 8$ using the average rate of change calculated in part i. an overestimate, an underestimate, or sometimes greater and sometimes less than the values given by the model function W? Explain your reasoning.

(c) Find $\lim_{t\to\infty} W(t)$. Interpret this value in the context of the problem.

3. The height of the tide along a shore is measured along a seawall. On a particular day, the water level at high tide was 13 feet, and occurred at midnight. At 6 a.m., the water level at low tide was 5 feet. Approximately every 12 hours, the cycle repeats.

   Suppose the sinusoidal function F models the water level along this shore, where $F(t)$ is measured in feet and t is measured in hours since midnight.

   (a) Provided below is a graph of F and its dashed midline. Five points, J, K, L, M, P, are labeled on the graph. No scale is indicated, and no axes are presented. Determine possible coordinates $(t, F(t))$ for J, K, L, M, P.



   (b) The function F can be written in the form

$$F(t) = a \sin(b(t + c)) + d$$

   Find the values of constants a, b, c, d.

   (c) Refer to the graph of F provided in part (a). Let $(t_1, F(t_1))$ and $(t_2, F(t_2))$ be the coordinates of L and M, respectively.

   i. On the time interval $t_1 < t < t_2$, is F increasing or decreasing?

   ii. Describe the change in the rate of change of F over $t_1 < t < t_2$.

4. (a) Consider the functions f and g, given by

$$f(x) = \frac{\csc x \sec^2 x - \csc x}{\sin x} \qquad g(x) = \log_4(x^3) + 3\log_8 x$$

i. Rewrite $f(x)$ as a single expression including only one trigonometric function.

ii. Rewrite $g(x)$ as a constant multiple of $\log_2 x$.

(b) Consider the functions h and k, given by

$$h(x) = \frac{e^5}{\sqrt{e^x}} \qquad k(x) = \arcsin(4x)$$

i. Solve the equation $h(x) = e$ for values of x in the domain of h.

ii. Solve the equation $k(x) = \frac{5\pi}{6}$ in the domain of k.

(c) The function p is given by

$$p(x) = 8\sec\left(\frac{\pi}{4} - x\right)$$

Find all values x in the domain of p which yield an output of $16\sqrt{3}$.